Based on K. H. Rosen: Discrete Mathematics and its Applications.

Lecture 1: Propositional logic. Section 1.1

1 Propositional logic

A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Example 1. "Clean up your room." is not a declarative sentence; hence, fails to be a proposition. On the other hand "$x + 2 = 3$ for $x = 5$" is a proposition.

Definition 2. Let p be a proposition. The negation of p is denoted by $\neg p$, is the statement "It is not the case that p." The proposition $\neg p$ is read "not p." The truth value of the negation of $\neg p$, is the opposite of the truth value of p.

Example 3. For p : "I like my CSI 30 class", the negation $\neg p$ will be "I do not like the CSI 30 class" or "It is not the case that I like the CSI class".

Definition 4. Let p and q be propositions. The conjunction of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction is true when both p and q are true and is false otherwise.

Definition 5. Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition "p or q". The disjunction is false when both p and q are false and is true otherwise.

Example 6. For p : "I am a BCC student" and q : "I lived in the Bronx", we have:

$$p \wedge q : \text{ I am a BCC student and lived in the Bronx"}$$

$$p \vee q : \text{ I am a BCC student or lived in the Bronx".}$$

The truth table for both $p \wedge q$ and $p \vee q$ are represented as:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The truth table for the exclusive or $p \oplus q$ and the implication $p \to q$ are:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Definition 7. Let p and q be propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

Definition 8. Let p and q be propositions. The conditional statement $p \to q$ is the proposition "if p, then q." The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \to q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence).

Example 9. The proposition If "Juan has a smartphone, then $2 + 3 = 5$" **is true independently of the truth value** of p : "Juan has a smartphone", since the conclusion $2+3 = 5$ is true. On the other hand the proposition "Juan has a smartphone, then $2 + 3 = 6$" is true if (and only if) Juan does not have a smartphone, even though $2 + 3 = 6$ is false.

The proposition $q \to p$ is called the converse of $p \to q$. The contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$. The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$. We will see that of these three conditional statements formed from $p \to q$, only the contrapositive always has the same truth value as $p \to q$.

Example 10. What are the contrapositive, the converse, and the inverse of the conditional statement
"The home team wins whenever it is raining?" Because "q whenever p" is one of the ways to express the conditional statement $p \to q$, the original statement can be rewritten as:
"If it is raining, then the home team wins."
Consequently, the contrapositive of this conditional statement is:
"If the home team does not win, then it is not raining."
The converse is:
"If the home team wins, then it is raining."
The inverse is:
"If it is not raining, then the home team does not win."
Only the contrapositive is equivalent to the original statement.

Definition 11. Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Questions

(a) Check that $p \to q$ is equivalent to $\neg p \lor q$.
(b) Show that the contrapositive of $p \to q$ has the same truth value as $p \to q$.